Problem Solving Across the Grades — Sample 2

Name:
Date:

Ten people are attending a party. Each person shakes hands with every one else at the party. Find two different ways to find the total number of handshakes.

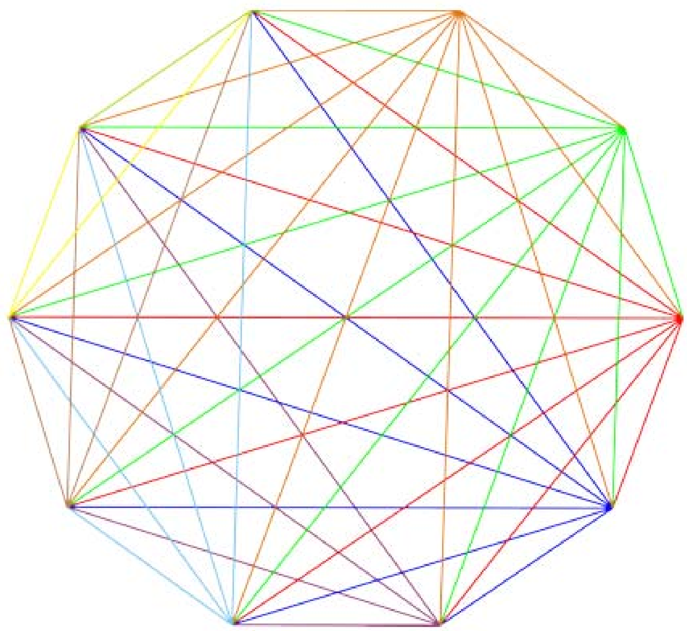
1.	2.

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1. Students can determine the total number of handshakes by drawing a diagram connecting each of the 10 students in the problem, then counting the number of lines very carefully.
This results in: $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ handshakes.

Problem-Solving Strategies:
- Draw a diagram or picture by hand or with the use of technology (e.g., The Geometer's Sketchpad® 4)



2. If students are working in a group, they might act the problem out, working with another group or groups, if necessary. A simple count would achieve the correct answer of 45.

Problem-Solving Strategies:
- Act it out

3. Some students might work this problem out logically, realizing that each of 10 students will shake the hand of 9 other students, i.e., $10 \times 9 = 90$. Further logical thinking will determine that this would include Student A shaking the hand of Student B, and Student B shaking the hand of Student A, so the total must be divided by 2 to get the correct answer of 45.

Problem-Solving Strategies:
- Use logical reasoning

Grades 7 and 8

4. Students might try looking at the same problem with a smaller number of students or by looking at a diagram (question 1) with fewer sides, then finding the pattern and extending it. The number of handshakes for up to 4 students could be determined fairly easily. (They might act it out in groups of 4.)

Problem-Solving Strategies:
- Make a simpler but similar problem
- Look for a pattern
- Make an organized list
- Act it out

Students might organize the data in a table of values.

Number of Students	Number of Handshakes
1	0
2	1
3	3 (1 + 2)
4	6 (1 + 2 + 3)

At this point some students will see a pattern (number of handshakes increasing by 1, 2, 3, …) and might continue the pattern.

Number of Students	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45

5. Some students may have been exposed to Pascal's Triangle and may recognize that the pattern they find if they model smaller problems of up to 4 or 5 handshakes is indeed in Pascal's Triangle. If they have a picture of the triangle in their text or on the classroom wall, they may refer to it to solve the problem.

Problem-Solving Strategies:
- Make a similar problem
- Find a pattern
- Recognize a pattern from a previous problem

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 51

6. Although it is not an expectation that students in these grades create a non-linear algebraic model, some students will determine the general term through inductive reasoning. They will (as noted above) notice that each of 10 students shakes hands with each of the other 9, and the result must be divided by 2 to get the correct answer. Hence, if n is the number of students, the number of handshakes can be determined by: $\frac{n(n-1)}{2}$.

Problem-Solving Strategies:
- Create an algebraic model
- Use logical reasoning

Students' solutions could include any of the Grades 7 and 8 answers.

Students might put the data in a table of values (see Grades 7 and 8 solutions) and check for first differences to see if the relation is linear or non-linear.

Problem-Solving Strategies:
- Find a pattern
- Make an organized list

Number of Students	Number of Handshakes	First Difference
1	0	
2	1	1
3	3	2
4	6	3

First differences are not constant, so the relation is non-linear.
Using first differences would make the pattern obvious and the table would be completed easily.

Number of Students	Number of Handshakes	First Difference
1	0	
2	1	1
3	3	2
4	6	3
5	10	4
6	15	5
7	21	6
8	28	7
9	36	8
10	45	9

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Problem-Solving Strategies:
- Find a pattern
- Make an organized list

1. Students look at second differences to determine if a relation is quadratic.

Number of Students	Number of Handshakes	First Difference	Second Difference
1	0		
2	1	1	1
3	3	2	1
4	6	3	1
5	10	4	

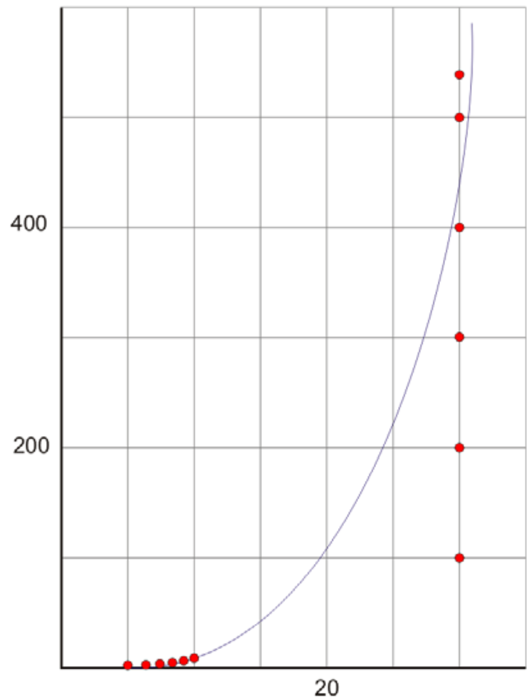
Second differences are constant so the relation is quadratic. Students complete the table.

Number of Students	Number of Handshakes	First Difference	Second Difference
1	0		
2	1	1	1
3	3	2	1
4	6	3	1
5	10	4	1
6	15	5	1
7	21	6	1
8	28	7	1
9	36	8	1
10	45	9	

Grade 10

2. If the question is extended for Grade 10 to a larger number, e.g., 30 students in a classroom, students might try to graph the relation using graphing software or a graphing calculator in order to find the number of handshakes. They could find the answer by finding an intersection with the line $x = 30$. This would also give them the value $y = 435$.

Problem-Solving Strategies:
- Make a graph
- Use technology
- Use algebra



Intersecting point is shown at (30, 435).

3. Students have become familiar with quadratic relations.

Students could determine the quadratic relation $T = \dfrac{n(n-1)}{2}$ using technology and apply the "formula."

Problem-Solving Strategies:
- Use technology
- Use algebra